HW 3 Solutions
EE 230

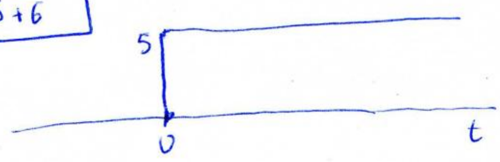
Problem 1

a) 1 pole at $s = -6 \Rightarrow T(s) = \frac{A}{s+6}$

DC gain of 9 $\Rightarrow T(j\cdot(\omega=0)) = T(s) = 9 \Rightarrow T(0) = \frac{A}{6} = 9 \Rightarrow A = 54$

Therefore $\boxed{T(s) = \frac{54}{s+6}}$

b) Step response to $5u(t)$



$x_{in}(t) = 5u(t) \Rightarrow X_{in}(s) = \frac{5}{s}$

$X_{out}(s) = X_{in}(s)\cdot T(s) = \frac{5}{s}\cdot\frac{54}{s+6} = \frac{270}{s(s+6)} = \frac{A}{s} + \frac{B}{s+6}$

$$\begin{cases} As + 6A + Bs = 270 \\ \Downarrow \\ As + Bs = 0 \end{cases} \Rightarrow \begin{cases} 6A = 270 \\ A = -B \end{cases} \Rightarrow \begin{cases} A = 45 \\ B = -45 \end{cases}$$

$\Rightarrow \boxed{X_{out}(s) = \frac{45}{s} - \frac{45}{s+6}}$

Take inverse Laplace Transform:

$x_{out}(t) = \mathcal{L}^{-1}\left(\frac{45}{s} - \frac{45}{s+6}\right) = \mathcal{L}^{-1}\left(\frac{45}{s}\right) - \mathcal{L}^{-1}\left(\frac{45}{s+6}\right)$

$\boxed{x_{out}(t) = 45u(t) - 45e^{-6t}u(t)}$

Problem 2

$$T(s) = 20\,\frac{s+2}{s^2+3s+9}$$

a) $s^2+3s+9 \Rightarrow P_1 = \dfrac{-3-\sqrt{3^2-4\times9\times1}}{2\times1}$; $P_2 = \dfrac{-3+\sqrt{3^2-4\times9\times1}}{2}$

$\Rightarrow P_1 = -\dfrac{3}{2} - \dfrac{\sqrt{-27}}{2}$; $P_2 = \dfrac{-3}{2} + \dfrac{\sqrt{-27}}{2}$

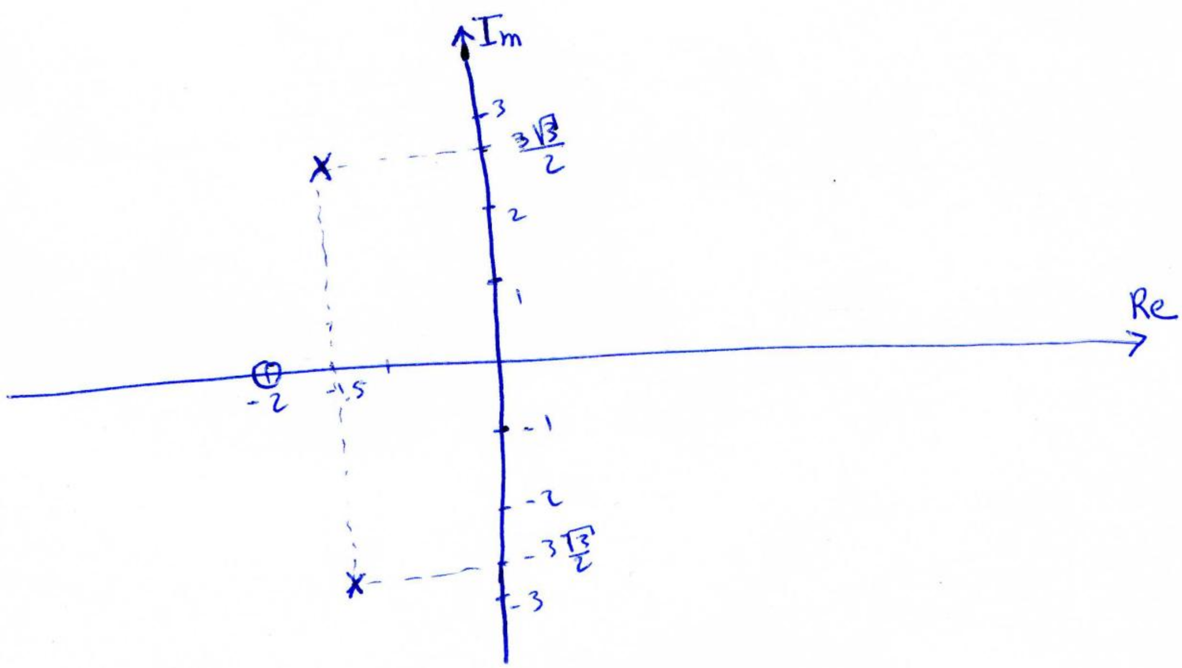
$\Rightarrow P_1 = \dfrac{-3}{2} - \dfrac{j3\sqrt{3}}{2}$; $P_2 = \dfrac{-3}{2} + \dfrac{j3\sqrt{3}}{2}$

$$\Rightarrow \boxed{T(s) = 20\,\frac{(s+2)}{\left(s+\frac{3}{2}+j\frac{3\sqrt{3}}{2}\right)\left(s+\frac{3}{2}-j\frac{3\sqrt{3}}{2}\right)}}$$

Zeros at $\cancel{z}$ $s=-2$

poles at $s = -\dfrac{3}{2} - j\dfrac{3\sqrt{3}}{2}$ and $s = \dfrac{-3}{2} + j\dfrac{3\sqrt{3}}{2}$

b)



Problem 3

a) $T(s) = 10 \dfrac{(s+1)(s-1)}{(s+8)(s-2+2j)(s-2-2j)}$

poles at $s = 2-2j$ and $s = 2+2j \Rightarrow \boxed{\text{unstable}}$

thes two poles are in the Right half of the s plane

b) $T(s) = 10 \dfrac{s^2+3s+18}{s^3+s^2+s+6}$

$s^3+s^2+s+6 = (s+2)(s^2-s+3) = (s+2)\left[s-\left(\frac{1+j\sqrt{11}}{2}\right)\right]\left[s-\left(\frac{1-j\sqrt{11}}{2}\right)\right]$

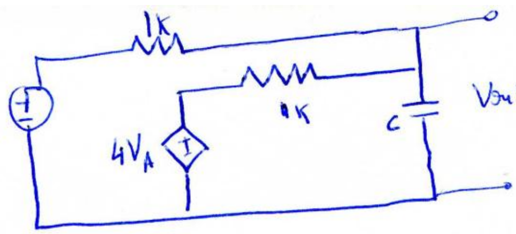
$\boxed{\text{The system is unstable}}$ becaus the poles $s = \frac{1+j\sqrt{11}}{2}$ and $s = \frac{1-j\sqrt{11}}{2}$ are in the Right half of the s-plane

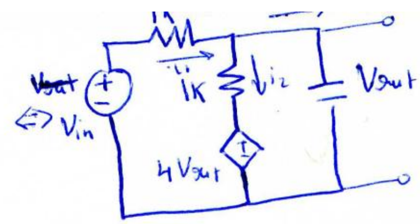
c) $T(s) = \dfrac{3}{s+3} + \dfrac{9}{s+20} - \dfrac{4}{s+1} = \dfrac{3(s+20)(s+1)+9(s+3)(s+1)-4(s+3)(s+2}{(s+3)(s+20)(s+1)}$

All the poles are in the Left Half of the plane $\Rightarrow \boxed{\text{stable}}$

d) $T(s) = \dfrac{10}{s+10}$ $\boxed{\text{stable}}$

Problem 4

a) 



Use KCL $\quad i_3 = i_1 - i_2$

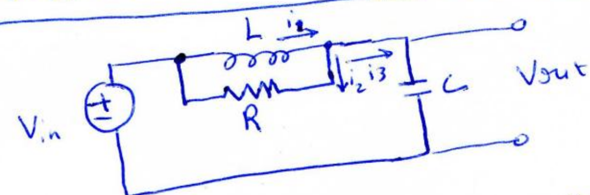
$$(S.C).\frac{V_{out}}{1} = \left(\frac{V_{in} - V_{out}}{1R}\right) - \left(\frac{V_{out} - 4V_{out}}{1R}\right)$$

$$\Rightarrow V_{out}\left(SC + \frac{1}{1R} + \frac{1}{1R} - \frac{4}{1R}\right) = \frac{V_{in}}{1R}$$

$$\Rightarrow T(s) = \frac{V_{out}}{V_{in}} = \frac{1}{1R\left(SC - \frac{2}{1R}\right)} = \frac{1}{1000SC - 2}$$

$$C = 1\mu F \Rightarrow \boxed{T(s) = \frac{1}{10^{-3}s - 2} = \frac{10^3}{s - 2000}}$$

$\boxed{\text{Pole at } s = 2000 \text{ in RHP} \Rightarrow \text{System unstable}}$

b) 

Use KCL $\quad i_3 = i_2 + i_1 = \left(\frac{V_{in} - V_{out}}{SL}\right) + \left(\frac{V_{in} - V_{out}}{R}\right) = (S.C)V_{out}$

$$\Rightarrow V_{out}\left(SC + \frac{1}{SL} + \frac{1}{R}\right) = V_{in}\left(\frac{1}{SL} + \frac{1}{R}\right)$$

$$T(s) = \frac{V_{out}}{V_{in}} = \frac{\frac{1}{SL} + \frac{1}{R}}{SC + \frac{1}{SL} + \frac{1}{R}} = \frac{1 + \frac{SL}{R}}{S^2LC + 1 + \frac{SL}{R}} = \frac{R + SL}{S^2RLC + R + SL}$$

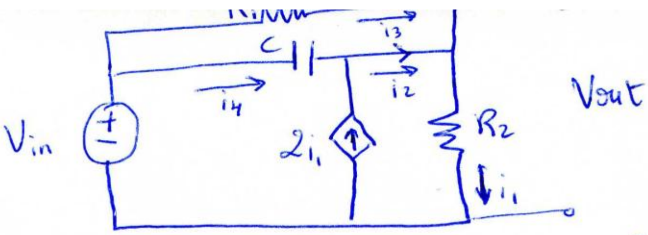
$$T(s) = \frac{2000 + 10^{-3}S}{S^2 \times 2\times10^3 \times 10^{-3} \times 5\times10^{-9} + 2\times10^3 + 10^{-3}s} = \frac{2000 + 10^{-3}s}{10^{-8}s^2 + 10^{-3}s + 2\times10^3} = \frac{2\times10^{11} + 10^5 s}{s^2 + 10^5 s + 2\times}$$

$$s^2 + 10^5 s + 2\times10^{11} = \left[s - (-5\times10^4 + j4.44\times10^5)\right]\left[s - (-5\times10^4 - j4.44\times10^5)\right]$$

$\boxed{\text{All poles in LHP} \Rightarrow \text{Stable}}$

Problem 4

c)



Use KCL $\begin{cases} i_1 = i_2 + i_3 & (1) \\ i_2 = 2i_1 + i_4 = 2i_1 + \left(\frac{V_{in} - V_{out}}{1}\right) s.C & (2) \end{cases}$

replace (2) in (1) $\Rightarrow i_1 = 2i_1 + (V_{in} - V_{out})sC + i_3$

$$\Rightarrow \frac{V_{out}}{R_2} = \frac{2V_{out}}{R_2} + (V_{in} - V_{out})sC + \frac{(V_{in} - V_{out})}{R_1}$$

$$\Rightarrow V_{out}\left(sC + \frac{1}{R_1} - \frac{1}{R_2}\right) = V_{in}\left(sC + \frac{1}{R_1}\right)$$

$$\Rightarrow T(s) = \frac{sC + 1/R_1}{sC + \frac{1}{R_1} - \frac{1}{R_2}} = \frac{sCR_1 + 1}{sCR_1 + 1 - \frac{R_1}{R_2}} = \frac{sCR_1R_2 + R_2}{sCR_1R_2 + R_2 - R_1}$$

$$sCR_1R_2 + R_2 - R_1 = 0 \Rightarrow s = \frac{R_1 - R_2}{CR_1R_2}$$

$R_1 = 1K$ and $R_2 = 5K$ $\Rightarrow$ $s < 0$ $\Rightarrow$ $\boxed{\text{system unstable}}$

Problem 5

$$T(s) = \frac{5}{s+2} \qquad X_{in}(t) = 4u(t)$$

a)

$$X_{in}(s) = \frac{4}{s}$$

$$X_{out}(s) = T(s)\,X_{in}(s) = \frac{20}{s(s+2)} = \frac{10}{s} - \frac{10}{s+2}$$

$$\boxed{x_{out}(t) = \mathcal{L}^{-1}\left(X_{out}(s)\right) = 10u(t) - 10e^{-2t}u(t)}$$

b) $$|T(s)| = |T(s)|_{s=j\omega} = \left|\frac{5}{j\omega+2}\right| = \frac{5}{\sqrt{4+\omega^2}}$$

$$|T(j\omega)| = \frac{1}{\sqrt{2}} \Rightarrow \frac{5}{\sqrt{4+\omega^2}} = \frac{1}{\sqrt{2}} \Rightarrow 50 = 4+\omega^2$$

$$\Rightarrow \boxed{\omega = \sqrt{46}\ \text{rad}}$$

Problem 6

$$T(s) = \frac{A}{s+5} = \frac{V_{out}}{V_{in}}$$

$$\Rightarrow V_{out}(s+5) = A V_{in}$$

$$\Rightarrow s V_{out} = A V_{in} - 5 V_{out}$$

let $A = 5$ $\Rightarrow$ $\boxed{s V_{out} = 5(V_{in} - V_{out})}$ (1)

Based on this circuit $sC V_{out} = \frac{(V_{in} - V_{out})}{R}$

or $\boxed{s V_{out} = \frac{V_{in} - V_{out})}{RC}}$ (2)

compare (1) and (2) $\Rightarrow$ $RC = \frac{1}{5}$

Choose $\boxed{R = 2k\Omega \quad C = 100\mu F}$

Problem 7

Pole at $S = -10$ and DC gain of $1/3$

$$T(S) = \frac{A}{S+10}$$

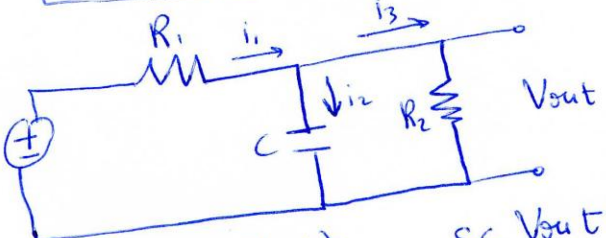
$$T(0) = \frac{1}{3} \Rightarrow \frac{A}{10} = \frac{1}{3} \Rightarrow A = 10/3$$

$$\boxed{T(S) = \frac{10/3}{S+10}} \quad ①$$

$$\cancel{\frac{V_{out}}{V_{in}} = \frac{(10/3)}{S+10} \Rightarrow V_{out}(S+10) = \frac{10}{3} V_{in}}$$

$$\cancel{\Rightarrow S.V_{out} = \frac{10}{3} V_{in} - 10 V_{out} = \frac{10}{3}(V_{in} - V_{out}) - \frac{20}{3} V_{out}}$$

$$\boxed{\cancel{S.V_{out} = \frac{10}{3}(V_{in} - V_{out}) - \frac{20}{3} V_{out}}} \quad ①$$



KCL
$$\frac{V_{out}}{R_2} = \frac{(V_{in} - V_{out})}{R_1} - SC\,V_{out}$$

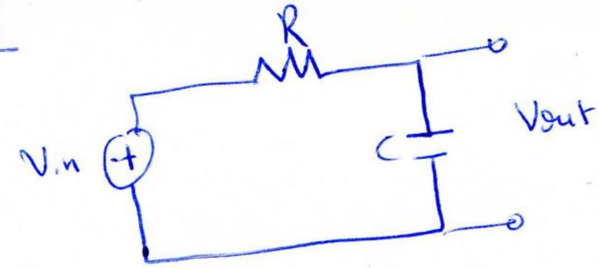
$$V_{out}\left(SC + \frac{1}{R_2} + \frac{1}{R_1}\right) = \frac{V_{in}}{R_1}$$

$$T(S) = \frac{V_{out}}{V_{in}} = \frac{1}{R_1\left(SC + \frac{1}{R_2} + \frac{1}{R_1}\right)} = \frac{R_2}{R_1 R_2 SC + R_1 + R_2} = \frac{\left(\frac{1}{R_1 C}\right)}{S + \left(\frac{R_1 + R_2}{R_1 R_2 C}\right)}$$

Compare to ① $\Rightarrow \begin{cases} \dfrac{1}{R_1 C} = \dfrac{10}{3} \Rightarrow R_1 C = 0.3 \\ \dfrac{R_1 + R_2}{R_1 R_2 C} = 10 \Rightarrow \dfrac{R_1 + R_2}{0.3 R_2} = 10 \Rightarrow 2R_2 = R_1 \end{cases}$

$\boxed{\text{Choose } C = 100\mu F \Rightarrow R_1 = 3k\Omega \text{ and } R_2 = 1.5k\Omega}$

Problem 8



$\cancel{T(s) = \frac{1}{RCs+1}}$

$\cancel{T}$ $V_{out} = \frac{1/sc}{R + \frac{1}{sc}} V_{in} \Rightarrow T(s) = \frac{V_{out}}{V_{in}} = \frac{1}{RCs+1}$

$w_c = \frac{1}{\sqrt{RC}} = 60\pi \text{ rad/s} \Rightarrow RC = 28 \times 10^{-6}$

$\boxed{\text{let } c = 10\,nF \Rightarrow R = \frac{28 \times 10^{-6}}{10 \times 10^{-9}} = 2.8\,k\Omega}$

1.39 (a) $A_v = \frac{v_o}{v_i} = \frac{10\text{ V}}{100\text{ mV}} = 100\text{ V/V}$

or, $20\log 100 = 40$ dB

$$A_i = \frac{i_o}{i_i} = \frac{v_o/R_L}{i_i} = \frac{10\text{ V}/100\ \Omega}{100\ \mu\text{A}} = \frac{0.1\text{ A}}{100\ \mu\text{A}}$$
$$= 1000\text{ A/A}$$

or, $20\log 1000 = 60$ dB

$$A_p = \frac{v_o i_o}{v_i i_i} = \frac{v_o}{v_i}\times\frac{i_o}{i_i} = 100\times 1000$$
$$= 10^5\text{ W/W}$$

or $10\log 10^5 = 50$ dB

(b) $A_v = \frac{v_o}{v_i} = \frac{2\text{ V}}{10\ \mu\text{V}} = 2\times 10^5\text{ V/V}$

or, $20\log 2\times 10^5 = 106$ dB

$$A_i = \frac{i_o}{i_i} = \frac{v_o/R_L}{i_i} = \frac{2\text{ V}/10\text{ k}\Omega}{100\text{ nA}}$$
$$= \frac{0.2\text{ mA}}{100\text{ nA}} = \frac{0.2\times 10^{-3}}{100\times 10^{-9}} = 2000\text{ A/A}$$

or $20\log A_i = 66$ dB

$$A_p = \frac{v_o i_o}{v_i i_i} = \frac{v_o}{v_i}\times\frac{i_o}{i_i}$$
$$= 2\times 10^5\times 2000$$
$$= 4\times 10^8\text{ W/W}$$

or $10\log A_p = 86$ dB

(c) $A_v = \frac{v_o}{v_i} = \frac{10\text{ V}}{1\text{ V}} = 10\text{ V/V}$

or, $20\log 10 = 20$ dB

$$A_i = \frac{i_o}{i_i} = \frac{v_o/R_L}{i_i} = \frac{10\text{ V}/10\ \Omega}{1\text{ mA}}$$
$$= \frac{1\text{ A}}{1\text{ mA}} = 1000\text{ A/A}$$

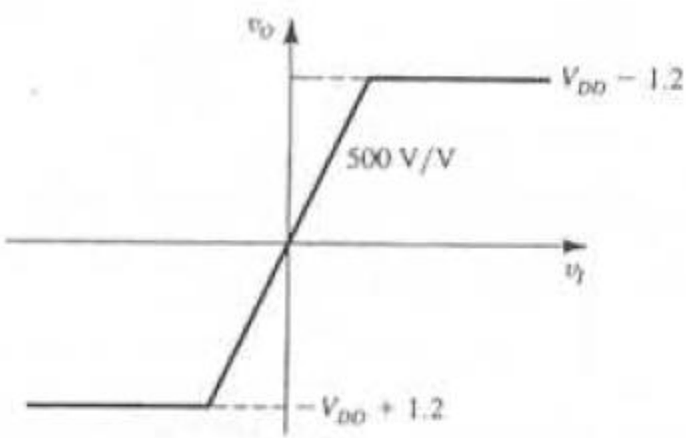
or, 20 log 1000 = 60 dB

$$A_p = \frac{v_o i_o}{v_i i_i} = \frac{v_o}{v_i} \times \frac{i_o}{i_i}$$
$$= 10 \times 1000 = 10^4 \text{ W/W}$$

or $10 \log_{10} A_p = 40$ dB

1.41 For $V_{DD} = 5$ V:
The largest undistorted sine-wave output is of 3.8-V peak amplitude or $3.8/\sqrt{2} = 2.7\,V_{rms}$. Input needed is 5.4 mV $_{rms}$.

Supplies are V_{DD} and $-V_{DD}$



For $V_{DD} = 10$ V, the largest undistorted sine-wave output is of 8.8-V peak amplitude or 6.2 V_{rms}. Input needed is 12.4 mV_{rms}.

For $V_{DD} = 15$ V, the largest undistorted sine-wave output is of 13.8-V peak amplitude or 9.8 V_{rms}. The input needed is 9.8 V/500 = 19.6 mV_{rms}.